Stochastic modeling of mesoscale eddies

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LU formalism

Stochastic material derivative

$$\frac{\mathbb{D}q}{\mathbb{D}t} = \frac{\partial q}{\partial t} + (\boldsymbol{u} + \dot{\boldsymbol{\eta}}) \boldsymbol{\cdot\nabla}\, q - \frac{1}{2}\, \boldsymbol{\nabla\cdot}\, \boldsymbol{\nabla\cdot}(\boldsymbol{a}q)$$

Noise and variance

$$\dot{\boldsymbol{\eta}}(\boldsymbol{x}, t) = \overline{\boldsymbol{\eta}}(\boldsymbol{x}) + \sum_k \boldsymbol{\phi}_k(\boldsymbol{x}, t)\underbrace{\xi_k}_{\sim \mathcal{N}(0,1)}, \;\; \boldsymbol{a} = \delta t \sum_k \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T$$

Stochastic evolution of layered PV

$$\frac{\mathbb{D}\boldsymbol{q}}{\mathbb{D}t} = F(\boldsymbol{q}) + D(\nabla^2 \boldsymbol{p}) + \dot{S}(\nabla^2 \boldsymbol{p})$$

QG kinematic relations

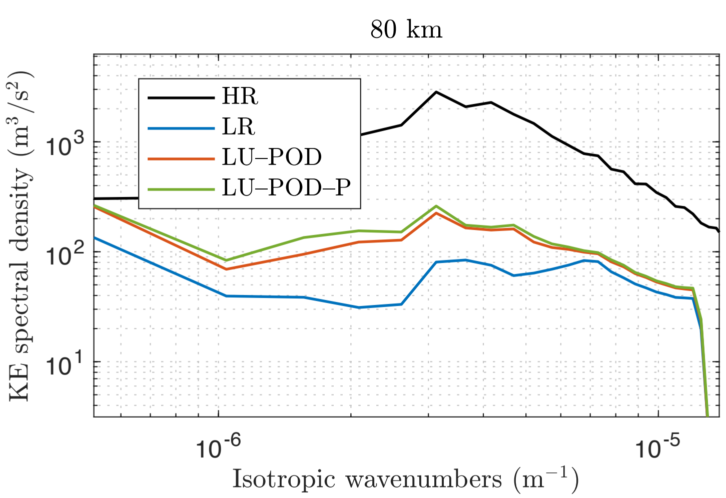
$$f_0(\boldsymbol{q} - \beta y) = \nabla^2 \boldsymbol{p} - f_0^2 \boldsymbol{A}\boldsymbol{p}, \;\; f_0 \boldsymbol{u} = \boldsymbol{\nabla}^{\perp} p$$

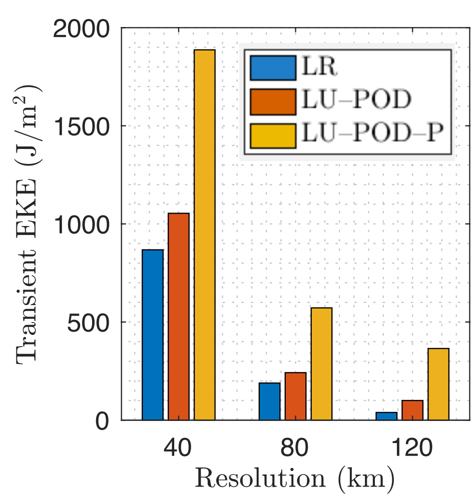
Parameterizations

LU–POD: $\overline{\boldsymbol{\eta}}$, $\{\boldsymbol{\phi}_k\}$ learned from data

LU–POD–P: $\boldsymbol{\phi}_k \perp \boldsymbol{\nabla}(\boldsymbol{A}\boldsymbol{p})$

Energy budget

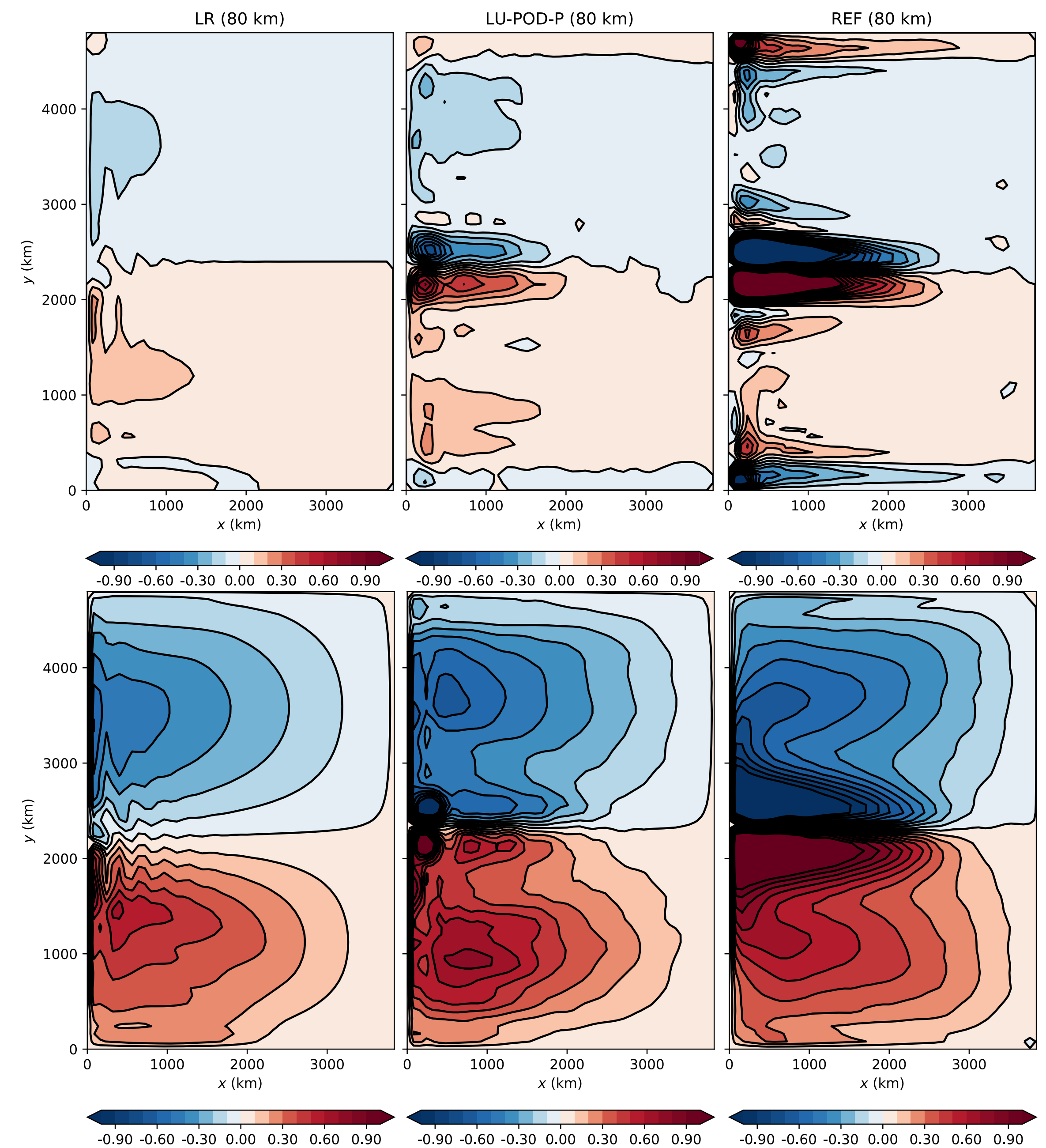




Benefits: reproduction of jet and recirculation, energy backscattering and internal variability

Long-term mean

Barotropic mode of $\boldsymbol{p}$



1st Baroclinic mode of $\boldsymbol{p}$

Reference: Li 2021, Stochastic modeling and numerical simulation of ocean dynamics, Chap. 8, https://hal.archives-ouvertes.fr/tel-03207741